Problem Set I: This problem set should be completed within 2 weeks.

1.) a.) Consider a weakly damped linear harmonic oscillator driven by white noise.

i.) Derive the fluctuation spectrum at thermal equilibrium.

ii.) What value of forcing is required to achieve stationarity at temperature *T*?

b.) Now consider a forced nonlinear oscillator

$$\ddot{x} + \gamma\dot{x} + \omega_0^2 x + \alpha x^3 = \tilde{f}.$$

Again, assume $\tilde{f}$ is white noise. Characterize the equilibrium fluctuation spectrum as completely as you can. Hint: You may find it useful to review Section 29 of "Mechanics", by Landau and Lifshitz.

2.) Consider a time-dependent phenomenological Ginzburg-Landau system, which satisfies

$$\frac{\partial\tilde{\eta}}{\partial t} - \Gamma_0 a\nabla^2\tilde{\eta} = -\frac{\widetilde{\delta H}}{\delta\eta} + \tilde{f},$$

$$H = +b(T - T_c)\frac{\eta^2}{2} + \frac{c\eta^4}{4},$$

where $\tilde{f}$ is white noise, in both space and time. Here $\tilde{\eta}$ is a fluctuation and η_0 is the mean value of the order parameter.

a.) Using the linear response, calculate the fluctuation spectrum and intensity for $\langle\tilde{\eta}^2\rangle$. Assume $T > T_c$, then consider $T < T_c$.

How does your result behave as:

i.) $k \to 0$ (large scale)

ii.) $T \to T_c$, from above (near criticality)

b.) Estimate when your calculation in a.) breaks down. Congratulations! - You have just re-derived the Ginzburg Criterion.

3.) a.) Derive the dispersion relation for a simple acoustic wave.

b.) Derive an energy theorem for the acoustic wave. Your theorem should have a structure similar to the Poynting theorem in Electromagnetism.

4.) Kulsrud 10.1

5.) Kulsrud 10.2

6.) Kulsrud 10.3

7.) Kulsrud 10.5